Tutorial

Faster than FFT: Conformal accelerations method

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Abstract: Fourier-Laplace transform technique allows one to represent several classes of important probability distributions and solutions of basic boundary problems for wide classes of fractional differential equations as integrals of functions enjoying two key properties: analytic continuation to a cone or the union of a cone and tube domain, and regular decay at infinity. Integral representations for Wiener-Hopf factors, fractional moments and special functions enjoy these properties as well. In the tutorial, we present the general methodology which allows one to evaluate the integrals enjoying these properties very fast and accurately. Among applications, we derive new efficient realizations of the Fourier, Laplace and Z-transforms, representations for probability distributions in Lévy models, stable ones including, and algorithms for pricing contingent claims, Monte-Carlo simulations, evaluation of special functions and filtering of highly persistent shocks.

Keywords: Operations research, Boundary problems for fractional differential equations, Lévy process, extrema of a Lévy process, barrier options, Wiener-Hopf factorization, Monte-Carlo simulations, Estimation and filtering, Fourier transform, Laplace transform, Z-transform

Topics covered in the tutorial:

(1) Types of integrals amenable to efficient evaluation using the conformal deformation techniques. Main groups of deformations and changes of variables. Error bounds and approximately optimal choice of the type of deformation and parameters of the change of variables.
(2) Evaluation of special functions (several examples).
(3) Pricing European options in Lévy models and solution of fractional-parabolic equations on $\mathbb{R}$. Probability distributions of Lévy processes. Efficient Fourier transform.
(4) Estimation and calilbration. Difficulties for methods based on FFT or fast Hilbert transform. Sundial calibration and ghost calibration.
(5) Efficient Wiener-Hopf factorization. Pricing options with barrier/lookback features and solution of boundary problems for fractional-differential operators. Efficient Laplace and Fourier transforms and examples of efficient deformations in double and triple integrals. Evaluation of distributions of an extrema of the Lévy process, joint distributions of an extrema and process, and two extrema and process. Examples of efficient deformations in triple integrals.
(6) Efficient evaluation of the probability distribution of a Lévy process, its extremum, and hitting time of the extremum. An example of efficient deformations in quintuple integrals.
(7) Simulation of distributions in (2), (5), (6).
(8) Simulation of multi-factor Lévy processes.
(9) Efficient Z-transform and applications to pricing barrier options with discrete monitoring, fractional moments and filtering of highly persistent shocks.