Abstract: Fourier-Laplace transform technique allows one to represent several classes of important probability distributions and solutions of basic boundary problems for wide classes of fractional differential equations as integrals of functions enjoying two key properties: analytic continuation to a cone or the union of a cone and tube domain, and regular decay at infinity. Integral representations for Wiener-Hopf factors, fractional moments and special functions enjoy these properties as well. In the tutorial, we present the general methodology which allows one to evaluate the integrals enjoying these properties very fast and accurately. Among applications, we derive new efficient realizations of the Fourier, Laplace and Z-transforms, representations for probability distributions in Lévy models, stable ones including, and algorithms for pricing contingent claims, Monte-Carlo simulations, evaluation of special functions and filtering of highly persistent shocks.

Keywords: Operations research, Boundary problems for fractional differential equations, Lévy process, extrema of a Lévy process, barrier options, Wiener-Hopf factorization, Monte-Carlo simulations, Estimation and filtering, Fourier transform, Laplace transform, Z-transform

Topics covered in the tutorial:

1. Types of integrals amenable to efficient evaluation using the conformal deformation techniques. Main groups of deformations and changes of variables. Error bounds and approximately optimal choice of the type of deformation and parameters of the change of variables.
5. Efficient evaluation of the probability distribution of a Lévy process, its extremum, and hitting time of the extremum. An example of efficient deformations in quintuple integrals.
7. Simulation of multi-factor Lévy processes.
8. Efficient Z-transform and applications to pricing barrier options with discrete monitoring, fractional moments and filtering of highly persistent shocks.
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